Geel 2000 Language Schools

Math Department

Second Term

Sec. 2

2023/2024

Name :-----------------------------------

Class:------------------------------------

Math. Department
Math.
second term
2nd secondary

Unit 1 : Sequences and series

Unit 1 : Lesson 1 : Sequences

1

Write down the general term for each of the following sequences:

a $(2, 5, 8, 11, ...)$ **b** $(\frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \frac{4}{5}, ...)$

2

Expand each of the following series, then find the expansion sum.

a $\sum_{r=1}^{4} (r^2)$ **b** $\sum_{r=1}^{7} (2r - 1)$ **c** $\sum_{r=1}^{n} (\frac{1}{r-1} - \frac{1}{r})$

3

Use the summation notation Σ to write down the series: $2 \times 3 + 3 \times 4 + 4 \times 5 + ...$

4

Find in two different methods $\sum_{r=1}^{4} (3 - 2r + r^2)$

Lesson 3 : Arithmetic sequences

Ex 1 :

Which of the following is an arithmetic sequence? why ?

a (7 , 10 , 13 , 16 , 19)

b (27 , 23 , 19 , 15 , 11, ...)

c $(\frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{5}, \frac{1}{6})$

Ex 2 :

In the arithmetic sequence (13 , 16 , 19 , ... , 100)

a Find the tenth term. **b** Find the number of the terms of the sequence.

Ex 4 :

Find the number of the terms of the arithmetic sequence (7 , 9 , 11 , ... , 65) then find the value of the tenth term from the end.

Ex 5 :

If the seventh and fifteenth terms of an arithmetic sequence are 18 and 34 respectively, find the common difference and the first term then find the n^{th} term of this sequence.

Ex 6 :

Find the arithmetic sequence whose sixth term = 17 and the sum of its third and tenth terms = 37.

Ex 7 :

Insert 5 arithmetic means between 6 and 48

Ex 8 :

Insert seven arithmetic means between the two numbers – 24 and 16

Ex 9 :

Find the order and value of the first negative term in the arithmetic sequence (67, 64 , 61 , ...)

Ex 10 :

Find the order and value of the first term whose value is greater than 180 in the arithmetic sequence:

Lesson 4 : Arithmetic series

Ex 1 :

Find $\sum_{r=5}^{24} (4r - 3)$

Ex 2 :

Find:

a $\sum_{K=1}^{20} (6K + 5)$

b $\sum_{m=7}^{32} (12 - 5m)$

Ex 3 :

In the arithmetic series 5 + 8 + 11 + ... find:

a The sum of its first twenty terms of the series .

b The sum of ten terms starting from the seventh term .

c The sum of the sequence terms starting from T_{10} up to T_{20}

Ex 4 :

In the arithmetic sequence (9 , 12 , 15 , ...), find :

a The sum of its first fifteen terms .

b The sum of the sequence terms starting from the fifth term up to the fifteenth term.

c The number of terms whose sum equals 750 starting from the first term .

Ex 5 :

Find the arithmetic sequence in which:

a $T_1 = 23$, $T_n = 86$, $S_n = 545$

b $T_1 = 17$, $T_n = -95$, $S_n = -585$

Ex 6 :

In the arithmetic sequence (25 , 23 , 21 , ...), find:

a The greatest sum of the sequence.

b The number of terms whose sum = 120 starting from the first term " Explain the existance of two answers".

Lesson 5 : Geometric sequences

Ex 1 :

Show which of the following sequences (T_n) is geometric , then find the common ratio of each :

a $(T_n) = (2 \times 3^n)$ **b** $(T_n) = (4\, n^2)$

c The sequence (T_n) where: $T_1 = 12$, $T_n = \frac{1}{4} \times T_{n-1}$ (where $n > 1$)

2

In the geometric sequence (2 , 4 , 8 , ...), find:

a The fifth term **b** the order of the term whose value is 512

3

(T_n) is a geometric sequence and all of its terms are positive. If $T_3 + T_4 = 6T_2$, $T_7 = 320$, find this sequence.

Ex 5 :

Find the geometric means of the sequence: (4 , ... , ... , ... , ... , ... , 2916)

Ex 6 :

Insert six geometric means between $\frac{1}{4}$ and 32

- **The relation between the arithmetic and geometric means of two numbers:**

Ex 7 :

If 6a , 3b , 2c , 2 d are positive quantities in an arithmetic sequence, prove that $b\,c > 2\,a\,d$

Lesson 6 : Geometric series

Ex 1 :

Find the sum of the geometric sequence in which : $a = 3$, $r = 2$, $n = 8$

Ex 2 :

Find the sum of the following two geometric sequences in which:

a $a = 4$, $r = 3$, $n = 6$ **b** $a = 1000$, $r = \frac{1}{2}$, $n = 10$

Ex 3 :

Find the sum of the geometric series : $1 + 3 + 9 + ... + 6561$

Ex 4 :

Find the sum of the following two geometric sequences:

a $a = 9$, $r = 3$, $\ell = 6561$ **b** $a = 2048$, $r = \frac{1}{2}$, $\ell = 128$

Using the Summation Notation

Ex 5:

Find $\sum_{r=5}^{12} 3\,(2)^{r-1}$

Ex 7 :

Which of the following series can you sum an infinite number of its terms ? Explain

a $75 + 45 + 27 + \ldots$ **b** $24 + 36 + 54 + \ldots$

Ex 8:

Find the sum for each of the following two geometric series if found:

a $\frac{81}{8} + \frac{27}{4} + \frac{9}{2} + \ldots$ **b** $\frac{2}{3} + \frac{5}{6} + \frac{25}{24} + \ldots$

Unit 2 : Permutations, Combinations

Lesson 1 : Counting Principle

Ex 1 :

The number of ways of sitting 4 students on 4 seats in a row equals :

Ex 2 :

How many three -digit numbers can be formed from the elements $\{2, 3, 5\}$?

Ex 3 :

How many different four - digit numbers can be formed from the elements $\{2, 3, 6, 8\}$ so that the unit digit is 6?

Lesson 2 : Permutations

Ex 1 :

a Find $\frac{\lfloor\underline{10}}{\lfloor\underline{8}}$ b If $\lfloor\underline{n} = 120$ find the value of n

Ex 2 :

Find: a $\frac{\lfloor\underline{15}}{\lfloor\underline{12}}$ b $\frac{\lfloor\underline{7}}{\lfloor\underline{5}} - \frac{\lfloor\underline{9}}{\lfloor\underline{7}}$

Ex 3 :

Find the solution set of the equation:- $\frac{\lfloor\underline{n}}{\lfloor\underline{n-2}} = 30$

Ex 4 :

Find the value for reach of the following:

a $^{7}p_{4}$ b $^{4}p_{4}$ c $^{4}p_{3}$

5

Calculate the value of the following:

a $^{5}p_{2} - {}^{6}p_{3}$ b $\frac{^{5}p_{5}}{^{5}p_{4}}$

Ex 6 :

Find the number of the different ways, for 5 students to sit on 7 seats in one row.

Ex 7 :

How many ways can 4 persons sit on 4 seats in the form of a circle ?

Ex 8 :

If ${}^{7}p_{r} = 840$, find the value of $\lfloor\underline{r-4}$

Ex 9 :

Find the value of the following:

a) $\lfloor\underline{7} \div \lfloor\underline{5}$

b) $3\lfloor\underline{2} - \lfloor\underline{3}$

c) ${}^{5}p_{3} \times \lfloor\underline{2}$

d) ${}^{3}p_{3} \times {}^{2}p_{2}$

e) ${}^{8}p_{1} - {}^{8}p_{2}$

f) ${}^{7}p_{0} \div {}^{7}p_{7}$

Lesson 3 : Combinations

Ex1 :

If ${}^{28}C_r = {}^{28}C_{2r-5}$, then find the value of r.

Ex 3 :

7 people have participated in a chess game so that a game is held between each two players. How many matches are there?

Ex 4 :

How many ways can a committee of two men and a woman be selected out of 7 men and 5 women?

Ex 5 :

If ${}^{n}C_3 = 120$, find the value of ${}^{n}C_{n-9}$

Unit 3 : Calculus

Lesson 1 : Rate of change

Ex 1 :

If $f(x) = 3x^2 + x - 2$

and x varies from 2 to 2 + h , find the function of variation V, then calculate the change in f when:

a h = 0,3

b h = - 0,1

2

If $f : [0, \infty[\longrightarrow \mathbb{R}$ where $f(x) = x^2 + 1$, find :

a The average rate of change function in f when $x = 2$, then calculate A (0.3)

b The average rate of change in f when x varies from 3 to 4

3

If $f(x) = x^2 - x + 1$, find the function of variation V when $x = 3$, then calculate:

a V(0.2)

b V(- 0.3)

Ex 4 :

If $f(x) = x^2 + 3x - 1$, find:

a The average rate of change function when $x = 2$, then find a (0.2)

b The average rate of change when x varies from 4.5 to 3

5

Find the rate of change function in f when $x = x_1$ for each of the following , then find this rate at the given values of x .

a $f(x) = 3x^2 + 2$ when $x = 2$

b $f(x) = \dfrac{2}{x - 1}$ when $x = 3$

6

Find the average rate of change function in f where $f(x) = \dfrac{3}{x - 2}$ when x varies from x_1 to $x_1 + h$, then deduce the rate of change in f when $x = 5$.

Lesson 2 : Differentiation

Ex 1 :

Find the slope of the tangent to the curve of the function f where $f(x) = 3x^2 - 5$ at point A (2, 7), then find the measure of the positive angle which the tangent makes with the positive direction of x-axis at point A to the nearest minute .

Ex 2 :

Find the slope of the tangent to the curve of the function f where $f(x) = x^3 - 4$ at point A (1, - 3), then find the measure of the positive angle which the tangent makes with the positive direction of x-axis at point A to the nearest minute.

Ex 3 :

Find the derivative function of the function f where $f(x) = x^2 - x + 1$ using the definition of the derivative, then find the slope of the tangent at the point (- 2 , 7)

4

If $f(x) = 3x^2 + 4x + 7$, find the derivative of the function f using the definition of the derivative, then find the slope of the tangent at the point (- 1 , 6)

Ex 5 :

Prove that $f(x) = \dfrac{x - 1}{x + 1}$ is differentiable when $x = 2$

Ex 6 :

Prove that $f(x) = x^2 - x + 1$ is differentiable when $x = 1$

7

Show that the function f where $f(x) = \begin{cases} x^2 & \text{when } x \leqslant 2 \\ x + 2 & \text{when } x > 2 \end{cases}$ is not- differentiable when $x = 2$

Ex 8 :

Discuss the differentiability of the function f at $x = 3$ where $f(x) = \begin{cases} 2x - 1 & \text{when } x < 3 \\ 7 - x & \text{when } x \geqslant 3 \end{cases}$

9

If the function f where $f(x) = \begin{cases} a\,x^2 + 1 & \text{when } x \leqslant 2 \\ 4x - 3 & \text{when } x > 2 \end{cases}$ is continuous at $x = 2$, find the value of the constant a , then discuss the differentiability of the function when $x = 2$

10

If $f(x) = a\,x^2 + b$ where a and b are two constants, find :

a The first derivative of the function f at any point (x, y).

b The two values of a and b if the slope of the tangent to the curve of the function at point $(2, -3)$ lying on it equals 12.

Lesson 3 : Rules of differentiation

Ex 1 :

Find $\frac{dy}{dx}$ in each of the following:

a $y = -3$ **b** $y = x^4$ **c** $y = 5x$

d $y = \frac{3}{x^2}$ **e** $y = \sqrt{x^3}$

Ex 2 :

Find $\frac{dy}{dx}$ in each of the following:

a $y = -\sqrt{2}$ **b** $y = \frac{4}{3}\pi\ x^3$ **c** $y = \frac{-4}{x^5}$ **d** $y = \sqrt[3]{x^5}$

Ex 3 :

Find $\frac{dy}{dx}$ in each of the following:

a $y = 2x^6 + x^{-9}$

b $y = \frac{\sqrt{x} - 2x}{\sqrt{x}}$

Ex 4 :

Find $\frac{dy}{dx}$ if:

a $y = 3x^8 - 2x^5 + 6x + 1$

b $y = \frac{5}{x} + x\sqrt{x} + \sqrt{3}\,x - 4$

5

Find $\frac{dy}{dx}$ if $y = (x^2 + 1)(x^3 + 3)$, then find $\frac{dy}{dx}$ when $x = -1$

Ex 6 :

Find $\frac{dy}{dx}$ if y = $(4x^2 - 1)(7x^3 + x)$, then find $\frac{dy}{dx}$ when $x = 1$

7

Find $\frac{dy}{dx}$ If $y = \frac{x^2 - 1}{x^3 + 1}$

8

Find $\frac{dy}{dx}$ if $y = \frac{x^3 + 2x^2 - 1}{x + 5}$

Ex 9 :

If $y = (x^2 - 3x + 1)^5$, find $\frac{dy}{dx}$

Ex 10 :

If $y = \sqrt[3]{z}$, $z = x^2 - 3x + 2$, find $\frac{dy}{dx}$

Ex 11 :

If $y = 3z^2 - 1$, $z = \frac{5}{x}$, find $\frac{dy}{dx}$

Ex 12 :

Find $\frac{dy}{dx}$ if

a $y = (6x^3 + 3x + 1)^{10}$

b $y = (\frac{x - 1}{x + 1})^5$

Ex 13 :

Find $\frac{dy}{dx}$ if $y = (\frac{5x^2}{3x^2 + 2})^3$

Ex 14 :

Find the values of x which make $f'(x) = 7$ in each of the following:

a $f(x) = x^3 - 5x + 2$

b $f(x) = (x - 5)^7$

Lesson 4 : Derivatives of trigonometric functions

Ex 1 :

Find $\frac{dy}{dx}$ for each of the following :

a $y = 5 \sin x$ **b** $y = x^3 \sin x$ **c** $y = 2 \sin (3x + 4)$

Ex 2 :

Find the first derivative for each of the following :

a $y = 2 \cos x - \tan 5x$ **b** $y = \tan (1 - x^2)$ **c** $y = \cos^2 (4x^2 - 7)$

Ex 3 :

find $\frac{dy}{dx}$ for each of the following :

a $y = 2 \tan 3x$ **b** $y = 2 \cos (4 - 3x^2)$ **c** $y = 2 \sin x \cos x$

d $y = 2x \tan x$ **e** $y = \tan^2 3x$ **f** $y = \tan 4x^3$

Lesson 5 : Applications on the derivative

Ex 1 :

Find the points which lie on the curve of $y = x^3 - 4x + 3$ at which the tangent makes a positive angle of measure $135°$ with the positive direction of x axis .

Ex 2 :

Find the points which lie on the curve of $y = x^2 - 2x + 3$ at which the tangent to the curve is :

a Parallel to x-axis

b Perpendicular to the straight line $x - 4y + 1 = 0$

Ex 3 :

Find the two equations of the tangent and normal to the curve of $y = 2x^3 - 4x^2 + 3$ at the point lying on the curve and whose abscissa = 2

Ex 4 :

Find the equation of the tangent to the curve of $y = 4x - \tan x$ at point $(\frac{\pi}{4}, f(\frac{\pi}{4}))$

Ex 5 :

If the curve $y = ax^3 + b\,x^2$ touches the straight line $y = 8x + 5$ at point $(-1, -3)$, find the two values of a and b.

Ex 6 :

Find the value of the two constants a and b if the slope of the tangent to the curve of $y = x^2 + a\,x + b$ at point $(1, 3)$ lying on it equals 5

Lesson 6 : Integration

Ex 1:

Prove that the function f where $F(x) = \frac{1}{2} x^4$ is an antiderivative to the function f where $f(x) = 2x^3$.

2

Show that the function F where $F(x) = \frac{1}{2} x^6$ is an antiderivative to the function f where $f(x) = 3x^5$

Ex 4 : Find :

a $\int x^5 \, dx$

b $\int x^{-3} \, dx$

c $\int x^{\frac{2}{5}} \, dx$

d $\int \frac{1}{\sqrt[5]{x^3}} \, dx$

Ex 5 :

Find:

a $\int x^8 \, dx$

b $\int x^{\frac{2}{3}} \, dx$

c $\int \sqrt[7]{x^5} \, dx$

d $\int 7x^{\frac{7}{9}} \, dx$

Ex 6 :

Find: **a** $\int (4x + 3x^2)\,dx$ **b** $\int \frac{(x^2+2)^2}{x^2}\,dx$

Ex 7 :

Find:

a $\int \left(2 + \sqrt{x} + \frac{1}{\sqrt{x}}\right) dx$ **b** $\int \left(\frac{1}{x^2} + \sqrt{x} + 3\right) dx$

Ex 8 :

Find:

a $\int \left((3 - 2x)^5 + 3) \right) \, dx$

b $\int \frac{x + 3}{(x - 2)^4} \, dx$

c $\int (x^2 - 3x + 5)^{-7} (2x - 3) \, dx$

d $\int (3x^2 - 2x + 1)^{11} (3x - 1) \, dx$

9

Find the following integrations:

a $\int (x - \sin x) \, dx$

b $\int (4 \cos x + \frac{1}{\cos^2 x} + 1) \, dx$

Ex 10 :

Find:

a $\int \cos(2x+3)\ dx$

b $\int (\sec^2 \frac{x}{2} - \sin(\frac{\pi}{4} - x))\ dx$

Ex 11 :

Find :

a $\int \sin(3x-5) f\ dx$

b $\int \cos(\frac{x}{3} - 2)\ dx$

Unit 4 : Trigonometry

Lesson 1 : Angles of elevation and depression

Ex 1 :

From a point on the ground surface a man observed the top of a tower at an angle of elevation of $20°$, He walked on a horizontal way in the direction of the tower base for 50 meters, the measurement of the angle of elevation of the tower top is $42°$. Find the height of the tower to the nearest meter .

From the top a rock of height 80 meters, the two angles of depression of the top and the base of a tower were measured to give $24°$ and $35°$ respectively. Find the height of the tower to the nearest meters known that the two bases of the rock and tower are in the same horizontal level.

Ex 4 :

From point A on a riverbank, a man observed the position of a home at point B on the other riverbank to find it in the direction of 20° North of the east. As he walks parallel to the riverbank in the direction of East for a distance of 300 meters to reach point C, he found point B in the direction of 46° North of the east. Find the width of the river to the nearest meter known that the two riverbanks are parallel and points A , B and C are at the same horizontal level.

Ex 5 :

A man measured the angle of elevation of a hill top from a point on the ground surface to find it 22°. As he ascends the hill for 500 meters on a road inclined to the horizontal by an angle of measurement 7°, he found the measure of the angle of elevation of the hill top is 64°. Find the height of the hill to the nearest meter .

Lesson 2 : Trigonometric functions of sum and difference of the measures of two angles

Ex 1 :

Find:

a $\sin 75^\circ$ b $\cos 15^\circ$ what do you notice?

Ex 2 :

Find.

a $\cos 105^\circ$ b $\sin 75^\circ \cos 15^\circ - \cos 75^\circ \sin 15^\circ$

c $\cos 80^\circ \cos 20^\circ - \sin 80^\circ \sin 20^\circ$

Ex 3 :

If $\sin A = \frac{3}{5}$ where $90^\circ < A < 180^\circ$, $\cos B = \frac{-5}{13}$

where $180^\circ < B < 270^\circ$

find $\cos(A - B)$, $\sin(A + B)$

4

In the triangle A B C, $\cos A = \frac{-3}{5}$ and $\sin B = \frac{5}{13}$, Find sin C without using the calculator.

Ex 5 :

Without using the calculator , prove that:

a $\tan 50^\circ = \dfrac{1 - \tan 5^\circ}{1 - \tan 5^\circ}$ **b** $\tan (45^\circ - A) = \dfrac{\cos A - \sin A}{\cos A - \sin A}$

Ex 6 :

If A, B and C are the measures of the angles of a triangle where $\tan B = \frac{4}{3}$, $\tan C = 7$, prove that $A = 45^\circ$

Ex 7 :

Find the solution set for each of the following equations where $0^\circ < x < 360^\circ$

a $\tan x + \tan 20^\circ + \tan x \tan 20^\circ = 1$ **b** $\sin (x + 30^\circ) = 2 \cos x$

Lesson 3 : The trigonometric functions of the double-angle

Ex 1 :

If you know $\sin A = \frac{4}{5}$ where $0° < A < 90°$,
find the value for each of the following without
using the calculator:

a $\sin 2A$ **b** $\cos 2A$ **c** $\tan 2A$

2

If $\cos A = \frac{4}{5}$, $0° < A < 90°$, find the values for each of the following without using the calculator:

a $\sin 2A$ **b** $\cos 2A$ **c** $\tan 2A$

3

Find the value for each of the following, without using the calculator , :

a $2 \sin 15° \cos 15$ **b** $2\cos^2 22° 30' - 1$

Ex 4 :

Find the value for each of the following Without using the calculator :

a $\sin \frac{\theta}{2}$ known that , $\sin \theta = -\frac{4}{5}$, $180° < \theta < 270°$

b $\cos 75°$

c $\tan 22° 30'$

5

Prove the correctness of the identity: $\csc 2x + \cot 2x = \cot x$, then use the previous identity to find the value of $\cot 15°$.

6

If $4 \cos 2C + 3 \sin 2C = 0$, find without using the calculator the value of $\tan C$, where C is the measurement of a positive acute angle.

7

Find the values of x included between 0 and 2π which satisfy the following equations:

a $\sin 2x = \sin x$

b $\cos^2 x - \sin^2 x = -\frac{1}{2}$

c $\tan^2 \frac{x}{2} + 2 \tan \frac{x}{2} = 1$

Lesson 4 : Heron's formula

Ex 1 :

Find the surface area of the triangle whose side lengths are 6, 8 and 10 centimetres using Heron's formula

Ex 2 :

Find the surface area of the triangle A B C in which:
a= 5cm , b = 12 cm , c = 13cm using Heron's formula.

Ex 3 :

Find the surface area of the triangle A B C in each of the following cases:

a) $a = 15cm$, $b = 12cm$, $c = 9$ cm

b) $b = 16cm$, $c = 20$ cm, $m(\angle A) = 60^{\circ}$

c) $a = 16cm$, $b = 18cm$, $c = 24$ cm

d $a = 32cm$, $b = 36$, $c = 30$ cm